International Journal of Theoretical Physics, Vol 4, No. 6 (1971), pp. 477–481

Proton–Neutron Mass Difference in Sudarshan's Theory of Universal Primary Interactions

R. ACHARYA

Department of Physics, Duke University, Durham, North Carolina 27706

H. H. ALY

Faculty of Physics, Southern Illinois University at Edwardsville, Edwardsville, Illinois 62025

and

K. SCHILCHER

Department of Physics, American University of Beirut, Beirut, Lebanon

Abstract

We have computed the neutron–proton mass difference in Sudarshan's Universal Theory of Primary Interactions by appealing to the Feynman–Speisman prescription. We predict the correct sign with $\delta M_p - \delta M_n \approx -0{\cdot}6$ MeV.

1. *Introduction*

Since the pioneering work of Feynman & Speisman (1954), which served to emphasize the connection between the sign of the proton–neutron mass difference and the dominance of the 'magnetic' energy over the 'coulomb' energy, there has been an endless stream of papers on the subject of the electromagnetic mass difference of neucleons.†

† Since these are too numerous to be listed here, we have chosen to list only a few and hopefully representative papers on the subject:

Cini, M., Ferrari, E. and Gatto, R. (1959). *Physical Review Letters*, **2**, 7.
Coleman, S. and Glashow, S. (1964). *Physical Review*, **134**, B671.
Dashen, R. (1964). *Physical Review*, **135**, B1196.
Wojtaszek, J., Marshak, R. E. and Riazuddin, (1964). *Physical Review*, **136**, B1053.
Acharya, R. and Narayanaswamy, P. (1966). *Physical Review*, **144**, 1305.
Pagels, H. (1966). *Physical Review*, **144**, 1261.
Barton, G. (1967). *Physical Review*, **153**, 1673.
Cohen, S. and Hagen, C. R. (1967). *Physical Review*, **157**, 1344.
Rockmore, R. M. (1967). *Physical Review*, **164**, 1929.

In the present note, we venture to suggest that the correct interpretation may actually be found in the original work of Feynman and Speisman, but reconsidered, however, in the light of the recent proposal of Sudarshan (1968) of the feasibility of a primary interaction theory involving the direct coupling of the observed vector and axial vector mesons to hadrons, leptons and photons. Sudarshan's theory has enjoyed quite a good measure of success in correlating various experimental results (Pradhan *et al.*, 1968; Chiang *et al.*, 1968; Srivastava and Tanaka, to be published). In a sense, it goes beyond all other known theoretical proposals† by seeking to extend the notion of universality of weak interactions so as to encompass the wider (and the ever widening) domain of weak, electromagnetic and strong interactions.

We have computed the electromagnetic self-energy of the n–p system according to the Feynman–Speisman prescription and employing the effective electromagnetic Lagrangian of Sudarshan's theory. We find that the magnetic energy indeed dominates over the Coulomb contribution, thus yielding the correct sign of the mass difference. This is taken as an indication of the soundness of Sudarshan's theory.

2. *Feynman–Speisman Prescription* (Feynman & Speisman, 1954)

The electromagnetic self mass of a nucleon is given to first order in the five-structure constant by the standard Feynman–Speisman expression:

$$\delta m_N = \frac{-i}{(2\pi)^4}\bar{U}(p)\int \frac{d^4k}{k^2}\left(g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right)\left[F_1(k^2)\gamma_\mu - \frac{i}{2m_N}F_2|(k^2)\sigma_{\mu\rho}k_\rho\right]$$
$$\times \frac{p-k+m_N}{k^2-2p.k}\left[F_1(k^2)\gamma_\nu + \frac{i}{2m_N}F_2(k^2)J_{\nu\tau}k_\tau\right]U(p) \qquad (2.1)$$

The effective electromagnetic Lagrangian of Sudarshan's theory reads (Sudarshan, 1968)

$$\mathscr{L}_{\text{e.m.}} = \bar{N}\left\{\frac{ge'm_\rho{}^2}{2(k^2-m_\rho{}^2)}\tau_3 + \frac{g_0e'm_\omega{}^2}{2(k^2-m_\omega{}^2)}\right\}\gamma_\lambda NA_\lambda$$
$$+\bar{N}\left\{\frac{g'e'm_\rho}{2(k^2-m_\rho{}^2)}\tau_3 + \frac{g_0{}'e'm_\omega}{2(k^2-m_\omega{}^2)}\right\}\sigma_{\lambda\nu}N\tfrac{1}{2}A_{\lambda\nu} \qquad (2.2)$$

where

$$e' = -\frac{e}{g}, \qquad g_0 = g, g' = \frac{5}{3}g \qquad \text{and} \qquad g_0{}' \approx 0{\cdot}04\,g \qquad (2.3)$$

† We have in mind the papers of:

Lee, T. D., Kroll, N. and Zumino, B. (1967). *Physical Review*, **157**, 1376.

Lee, T. D. and Zumino, B. (1967). *Physical Review*, **163**, 1667.

Lee, T. D. (1968). *Physical Review*, **171**, 1731.

See also, Lee, T. D., Weinberg, S. and Zumino, B. (1967). *Physical Review Letters*, **18**, 1029.

Equation (2.2), with the choice $m_\rho = m_\omega$ yields the identifications:

$$F_1(k^2) = -e\frac{m_\rho^2}{k^2 - m_\rho^2} \qquad \text{proton}$$
$$= 0 \qquad \text{neutron} \tag{2.4}$$

and

$$F_2(k^2) = -e\frac{2m_N m_\rho G_{p,n}}{k^2 - m_\rho^2}$$

where

$$G_{p,n} = \pm\frac{g' + g_0'}{2g} \tag{2.5}$$

The form factors displayed in Equation (2.4) may be compared with the choice of Feynman & Speisman (1954). They appear there under the disguise of regulator functions. Feynman and Speisman discovered that if one chooses a high enough cut-off, the extra factors in the momentum arising from the derivative coupling of the anomalous moment to the photon will dominate over the coulomb contribution yielding

$$\delta M \equiv \delta M_p - \delta M_n < 0$$

Equations (2.1) and (2.4) yield the finite (infrared divergence-free) result

$$\begin{aligned}\delta m_p = \frac{\pi^2 e^2}{(2\pi)^4} m_N \alpha \Bigg\{&\left[3 - \frac{\alpha}{2}\ln\alpha + (3\alpha^2 - 10\alpha + 4)(\tan^{-1}/\Delta)\right] \\ &+ 6G_p \alpha^{1/2}[-\tfrac{1}{2}\ln\alpha + (\alpha - 2)(\tan^{-1}/\Delta)] \\ &+ G_p^2\left[1 - \frac{3+\alpha}{2}\ln\alpha + (\alpha^2 + \alpha - 8)(\tan^{-1}/\Delta)\right]\Bigg\}\end{aligned} \tag{2.6}$$

$$\delta m_N = \frac{\pi^2 e^2}{(2\pi)^4} m_N \alpha G_N^2 \left\{1 - \frac{3+\alpha}{2}\ln\alpha + (\alpha^2 + \alpha - 8)(\tan^{-1}/\Delta)\right\} \tag{2.7}$$

where

$$\alpha = \frac{m_\rho^2}{m_N^2}; \qquad \tan^{-1}/\Delta \equiv \frac{1}{\sqrt{[\alpha(4-\alpha)]}}\left[\tan^{-1}\frac{\alpha}{\sqrt{[\alpha(4-\alpha)]}} + \tan^{-1}\frac{2-\alpha}{\sqrt{[\alpha(4-\alpha)]}}\right] \tag{2.8}$$

Equations (2.6) and (2.7) give

$$\delta m \equiv \delta m_p - \delta m_n \simeq -0{\cdot}6 \text{ MeV} \tag{2.9}$$

to be compared with the experimental value

$$\delta m \simeq -1{\cdot}3 \text{ MeV}$$

Hence, the correct sign is obtained, although the numerical value of the prediction is considerably below the observed value. As one increases the ratio g_0'/g upward, the numerical value of $|\delta M|$ increases rapidly, but only at the expense of the magnetic moment predictions. Certainly, one can not (and should not) expect *quantitative* agreement of the theory in *all* areas of physical phenomena.

3. *Discussion and Conclusion*

We have found that with the choice of the form factors dictated to us by Sudarshan's theory, one indeed recovers the correct *sign* of the neutron–proton mass difference. We are, of course, aware of the fact that the choice of $F_{1,2}(k^2)$ made here disagrees with experimentally observed dipole behaviour of the form factors. We have no convincing argument to explain this apparent discrepancy†.

The concept of feedback (Fried & Truong, 1966) has played no role in our analysis. In view of Barton's criticism‡ (Barton & Dare, 1966), we believe that a sign reversal of the 'feedback type' must be looked upon with great suspicion. On the other hand, the sign reversal of the 'driving type',§ if it is operative, will only serve to enhance the numerical value of our prediction, without disturbing the sign.

In conclusion, we wish to emphasise that our calculation of the n–p mass difference should only be viewed in the limited sense of having successfully implemented the Feynman–Speisman idea within the framework of Sudarshan's theory with the important difference that physically observed vector meson masses have replaced the arbitrary cut-offs of Feynman & Speisman (1954).

Acknowledgement

We would like to thank E. C. G. Sudarshan, G. Barton and C. Ryan for their very valuable discussions.

† See, however, the remarks of Sudarshan (1968).

‡ Incidentally, we are, of course, aware of the opinion expressed in the literature that one should not seriously consider tackling the problem of $\Delta I = 1$ mass differences without introducing strange particles. But our object here has been only to demonstrate that the correct sign of the n–p mass difference is obtained in Sudarshan's theory.

§ By a sign reversal of the 'driving type', we mean $\delta M = \delta M^{\gamma} + \chi\delta M$ where δM^{γ} is the Feynman–Speisman contribution and $x < 1$. Clearly, a sign reversal of the 'feedback' variety with $x > 1$ will tend to reverse the initial sign of δM^{γ}. This would be, of course, disastrous if $\delta M^{\gamma} < 0$, as is the case here. It is here that Barton's criticism is very relevant (see Barton & Dare, 1966, and also Cohen & Hagen, (1967): A sign reversal of the feedback type ($x > 1$) cannot be taken seriously in an elementary particle picture, since the mass shift δM has gone through a singularity at $x = 1$. On the other hand, a sign reversal of the driving type ($x < 1$) is perfectly consistent with our result $\delta M^{\gamma} < 0$ and, in fact, it will tend to improve numerical agreement with experiment.

References

Barton, G. and Dare, D. (1966). *Physical Review*, **150**, 1220.
Chiang, C., Gleiser, R., Huq, M. and Saxena, R. (August, 1968). NYO-3399-168 and SU-1206-168.
Cohen, S. and Hagen, C. R. (1967). *Physical Review*, **157**, 1344.
Feynman, R. and Speisman, G. (1954). *Physical Review*, **94**, 500.
Fried, H. and Truong, T. N. (1966). *Physical Review Letters*, **16**, 559.
Pradhan, T., Sudarshan, E. C. G. and Saxena, R. P. (1968). *Physical Review Letters*, **20**, 79.
Srivastava, P. P. and Tanaka, K. (to be published).
Sudarshan, E. C. G. (1968). *Proceedings of the Royal Society*, **A305**, 319.